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Analysis of Cracks at Attachment Lugs

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An analytical procedure using the finite-element method with the inclusion of a high-order crack-tip singularity element was used to determine the pin-bearing pressure distributions and the stress intensity factors for cracks in both straight and tapered lugs. Other parameters evaluated include the crack length, the outer-to-inner radius ratio, and the relative rigidity of the pin and the lug. Based on this study, it is found that the pin-bearing pressure distribution changes with the change in crack length. In the presence of a crack, the pressure distributions change significantly compared with the uncracked case, and its associate stress intensity factor is sensitive to the pin-bearing pressure distributions, especially when the crack length is large or the outer-to-inner radius ratio is small. The effect of the relative rigidity of the pin and the lug on the stress intensity factor is negligible.

Nomenclature

a = crack length
K = opening mode stress intensity factor
K_t = stress concentration factor
P = total applied force
R_i = hole radius of lug
R_o = outer radius of the circular head
t = thickness of lug
W = width of the shank of lug
σ_0 = far-field gross section stress, P/Wt
σ_{br} = average bearing stress, $= P/2R_it$

Introduction

In aircraft structures, lug-type joints are used frequently in linkage structure or to connect major structural components. The lug joint is normally connected by a single bolt or pin, creating a simple joint that is easy to assemble and disassemble. Since clamping of the joint is not normally allowed, the lug can act as a pivot. However, the elastic gross section stress concentration for normal lugs is very high, and this high stress concentration at the hole edge makes for a relatively short crack initiation/crack growth life.

Since attachment lugs are some of the most fracture-critical components in aircraft structure, the consequences of a structural lug failure can be very severe. Therefore, it is important to develop an analytical procedure for assessing or designing durable attachment lugs to help ensure aircraft operational safety.

During the past decade, the influence of fracture mechanics on the design, manufacture, and maintenance of aircraft has steadily increased, and nondestructive inspection techniques have been improved significantly. However, some cracks still cannot be detected during routine maintenance inspection. Under service loading, such cracks will grow and fracture can occur if the crack length reaches a critical dimension before it can be detected and the part repaired or replaced. To assure aircraft safety, the U. S. Air Force has imposed damage-tolerance requirements (MIL-A-83444)[1] which include the prediction of fatigue crack growth life and residual strength of the structure by assuming that small initial cracks exist at critical locations of new structure due to various material and manufacturing and processing operations. Life prediction requires the determination of stress intensity factors, which depend upon the complexities of structural arrangements, crack geometry, and applied loads.

Athough the fatigue of straight and tapered attachment lugs has been studied extensively,[2-9] only limited analyses have been conducted to determine the stress intensity factors K for through cracks in straight lugs. Schijve and Hoeymakers[10] and Wanhill[11] derived empirical K solutions from the growth rate data of through cracks under constant-amplitude loading using a back-tracking method such as that proposed by James and Anderson.[12] Analytically, Liu and Kan[13] and Kirkby and Rooke[14] used the simple compounded solution method which involves superimposing known solutions such as that in Ref. 15 to estimate the stress intensity factors. Aberson and Anderson[16] used a special crack-tip singularity element to compute the stress intensity factors for a crack in a nonsymmetrical aft lug of an engine pylon. Pian et al.[17] used the hybrid finite-element method to compute the K values for cracks oriented in various angles from the axial direction of straight lugs. Impellizzeri and Rich[18] modified the exact weight function, derived by Buecker[19] for an edge crack in a semi-infinte plate to include a series of geometry correction factors and computed the K values using the weight function method. Except Ref. 16, all of these studies made the assumption that the assumed or computed pin-bearing pressure distribution for an uncracked case remains unchanged, even after the crack has initiated and propagated. Based on the parametric study conducted in Ref. 17, it was found that, for any given crack length, the difference in the stress intensity factor computed using the uniform and cosine pin-bearing pressure distributions was as much as 30%. Therefore, it is obvious that the correct representation of the pin-bearing pressure distribution during the crack growth process is essential to the calculation of accurate stress intensity factors.

The purpose of the present study is to investigate the effect of the crack length on the pin-bearing pressure distribution, and to compute the stress intensity factors for various cracked attachment lugs loaded through a neat-fit pin. The effect of different fitting, such as clearance and interference, on the stress intensity factor is currently under study, and the result will be reported in the near future.

Analysis and Discussion

The finite-element method with the inclusion of a high-order crack-tip singularity element was used to calculate the pin-bearing pressure distribution and the stress intensity factors for single and double cracks emanating from straight and tapered attachment lugs. Figure 1 shows a typical model, which was used for a single crack emanating from a tapered

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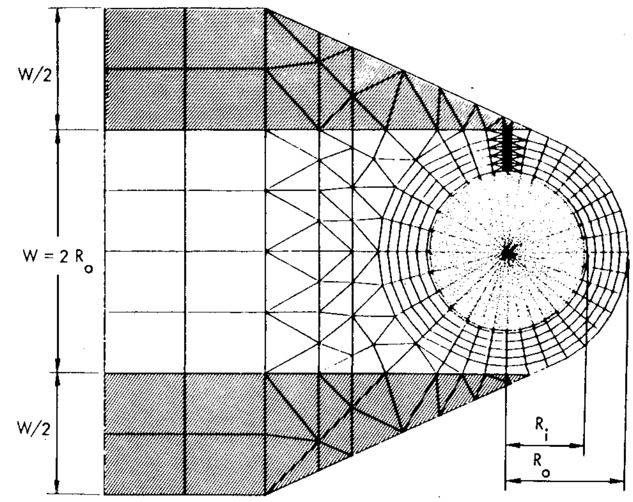

Fig. 1 Finite-element model.

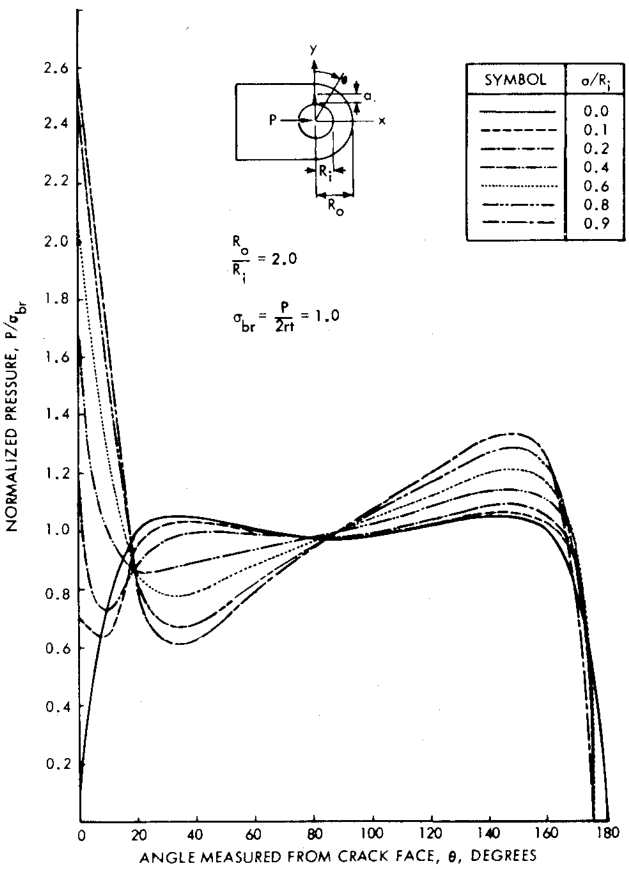

Fig. 2 The pin-bearing pressure distributions for single cracks at a straight attachment lug.

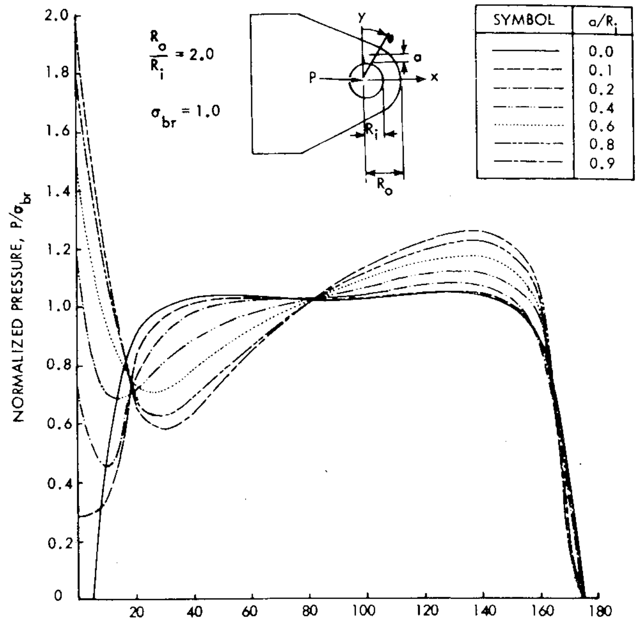

Fig. 3 The pin-bearing pressure distributions for single cracks at a tapered attachment lug.

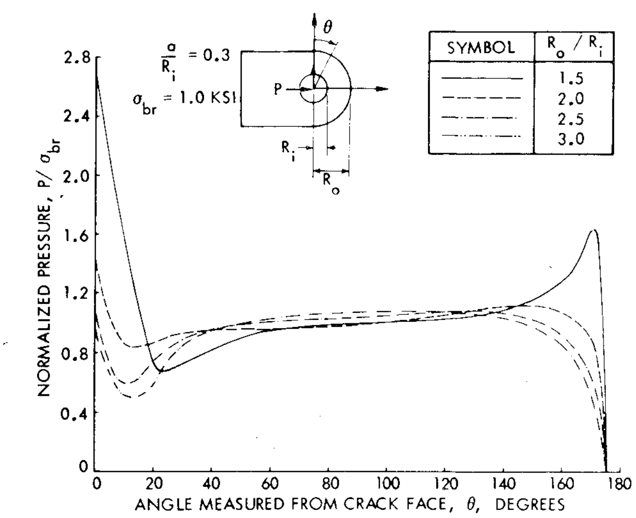

Fig. 4 Bearing pressure distributions at straight attachment lugs having various R_o/R_i ratios and $a/R_i = 0.3$.

lug having an outer-to-inner radius ratio, R_o/R_i, of 1.5. The angle between two edge surfaces of tapered head is 50 deg. For a straight lug study, the model was modified by removing the shaded elements on both sides of a tapered lug. The width of a tapered lug is twice the width of a straight lug. For a double-crack case, due to symmetry, only the upper half of the model was used. Constant-strain triangular and quadrilateral elements were used to represent the region away from the crack tip. A 10-node, high-order singularity element, developed at Lockheed-Georgia,[16] was used at the crack tip region for computing the stress intensity factors appropriate to several crack lengths, and the pin was modeled by a series of solid triangular elements as shown in the dotted area. Spring elements were used to connect the pin and lug at each pair of nodes having identical nodal coordinates along the contact surface.

Contact conditions were determined through the iterative analysis by removing the spring elements which were in tension. To load the model, a concentrated force was applied at the center of the pin and reacted at the other end of the lug. This approach was used to determine the pressure distribution on the pin-lug contact surface and the stress intensity factors for cracks having normalized crack lengths, $a/(R_o - R_i)$, ranging from 0 to 0.9. General parameters evaluated included the type of lug, the outer-to-inner radius ratio (R_o/R_i), and the relative rigidity of the pin and the lug.

Typical pin-bearing pressure distributions obtained for single cracks emanating from steel attachment lugs loaded by the neat fitting steel pins are presented in Figs. 2-5. Figures 2 and 3 show the effect of crack length on the pin-bearing pressure distributions for a straight lug and a tapered lug, respectively, each having an R_o/R_i ratio of 2.0. Figures 4 and 5 show the effect of variations in the outer-to-inner radius ratio on the pressure distributions of such lugs having a constant a/R_i ratio of 0.3. From Figs. 2 and 3, one can see

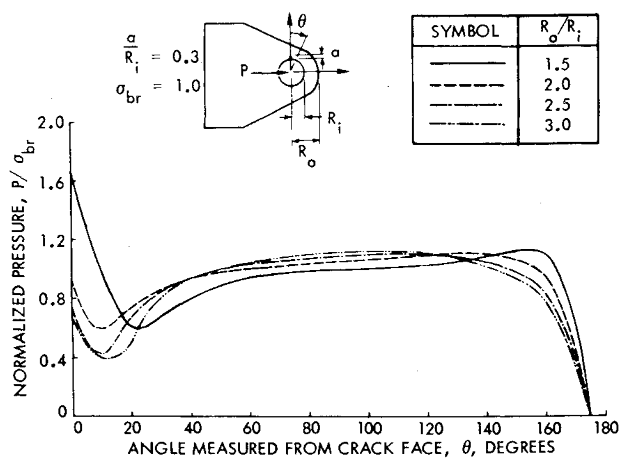

Fig. 5 Bearing pressure distributions at tapered attachment lugs having various R_o/R_i ratios and $a/R_i = 0.3$.

that, when there is no crack, the initial shape of the distribution is close to cosine at edges and uniform at the central portion of the contact surface. As the crack length increases, the pressure distribution changes, especially at the location adjacent to the crack face. At the crack mouth ($\theta = 0$ deg), the magnitude of pressure increases rapidly as the crack length increases. For example, the pressure increases from zero for $a/R_i = 0$ to more than twice the average pressure for $a/R_i \geq 0.6$. The pressure distribution along the contact surface also changes with the crack length. It decreases from an initial maximum value at the crack mouth to a local minimum and then gradually increases again and approaches another maximum before decreasing to zero at the end of the contact surface. The location of the local minimum pressure changes with the change of crack length. It changes from 0-deg location (measured from the crack face) when the crack length is very small to about 30-deg location when the crack length exceeds 6/10 of the radius. The results shown in Figs. 4 and 5 indicate that, for a constant normalized crack length a/R_i, the change of pin-bearing pressure distribution is most significant when the R_o/R_i ratio is small. When the R_o/R_i ratio increases, the variation in the pressure distribution decreases. The variation in pressure distribution is smaller for a tapered lug than for a straight lug.

The effect of pin-bearing pressure distributions on stress intensity factors is shown in Fig. 6 for single cracks in a straight lug having an R_o/R_i ratio of 1.5. In Fig. 6, the circle and triangle symbols are results obtained from Ref. 17,† where it is assumed that the pin-bearing pressure distributions are cosine and uniformly distributed along the 180-deg contact surface, respectively, and that such distribution remains unchanged with the crack length. The square symbol represents the results obtained using the present analysis, which has properly accounted for the change in pressure distribution as the result of crack extension. As can be seen from this figure, when the crack is small ($a/R_i < 0.05$), the current computed K value is practically the same as that obtained using a uniform pin-bearing pressure distribution. However, as the crack length increases, the current analysis gives a lower K value than the others. This is because when the crack length increases, the pressure near $\theta = 0$ deg increases markedly and exceeds the average pin-bearing pressure (see Fig. 2). This high pin-bearing pressure, when applied near $\theta = 0$ deg in the direction almost parallel to the crack orientation, intends to close the crack surfaces, hence reducing the effective stress intensity factor as discussed by Brussat.[20]

The computed normalized opening mode stress intensity factors using the steel pin and the steel lug model are presented in Fig. 7 as a function of normalized crack length

†The results are mislabeled in Ref. 17; uniform distribution results are labeled cosine distribution and vice versa.

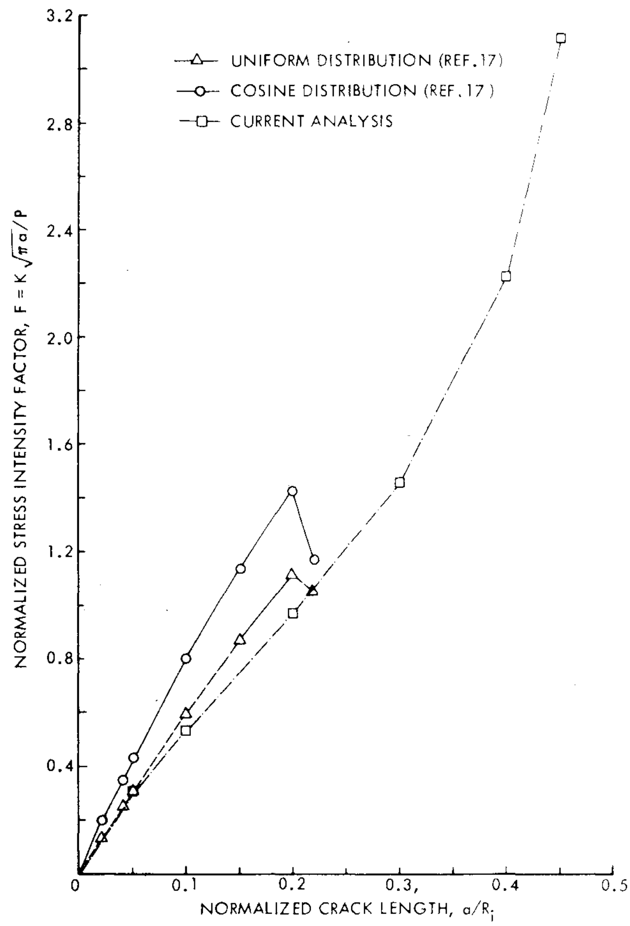

Fig. 6 Sensitivity of stress intensity factors to pressure distribution, $R_o/R_i = 1.50$.

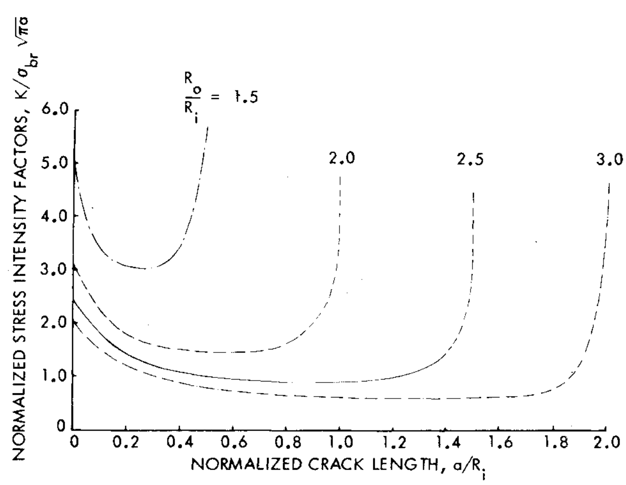

Fig. 7 Normalized stress intensity factors for single crack emanating from straight attachment lugs.

for single cracks emanating from the hole wall of straight attachment lugs with the outer-to-inner radius ratio, R_0/R_i, ranging from 1.5 to 3.0. Similar results obtained for tapered attachment lugs are shown in Fig. 8. In all cases, the computed sliding-mode stress intensity factors are much smaller than those of opening mode. Hence, they are not presented in the figures. It should be noted that the K values were normalized in terms of the average bearing stress σ_{br} instead of far-field gross section stress σ_0 for the convenience of presenting uniformly the results obtained for both straight and tapered lugs. To convert these normalized factors in terms of σ_0, one can simply multiply these normalized factors by the ratio of R_o/R_i for straight lugs and $2R_o/R_i$ for tapered

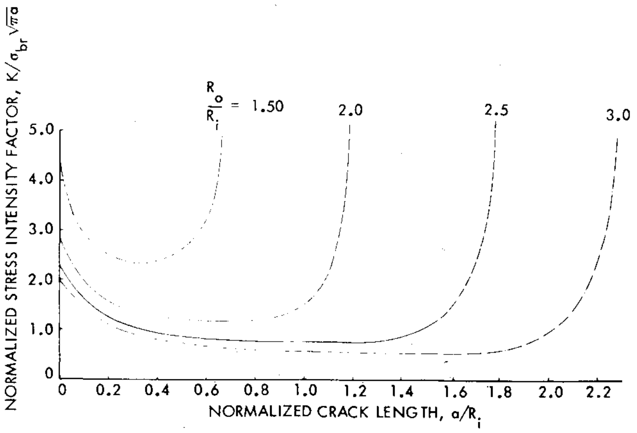

Fig. 8 Normalized stress intensity factors for single crack emanating from tapered attachment lugs.

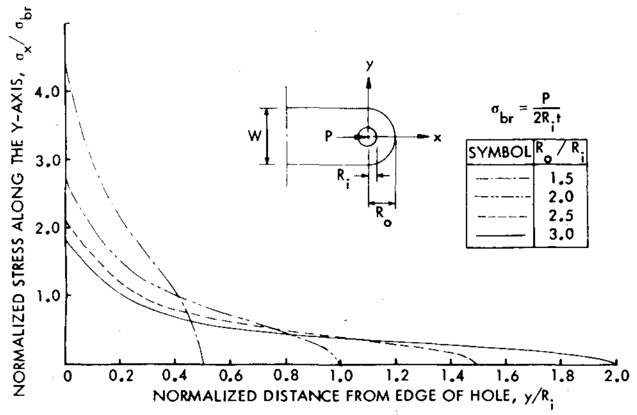

Fig. 9 Unflawed stress distributions at straight attachment lugs having various R_o/R_i ratios.

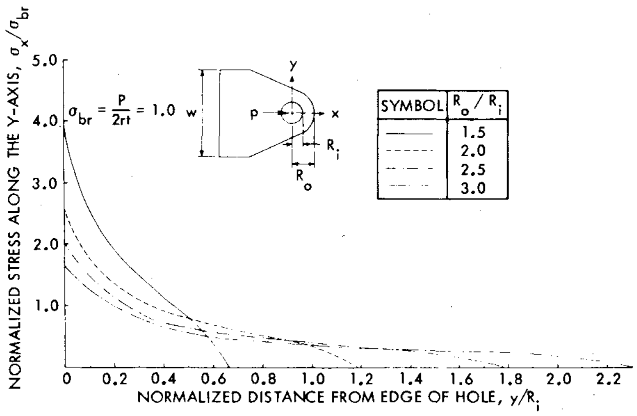

Fig. 10 Unflawed stress distribution at tapered attachment lugs having various R_o/R_i ratios.

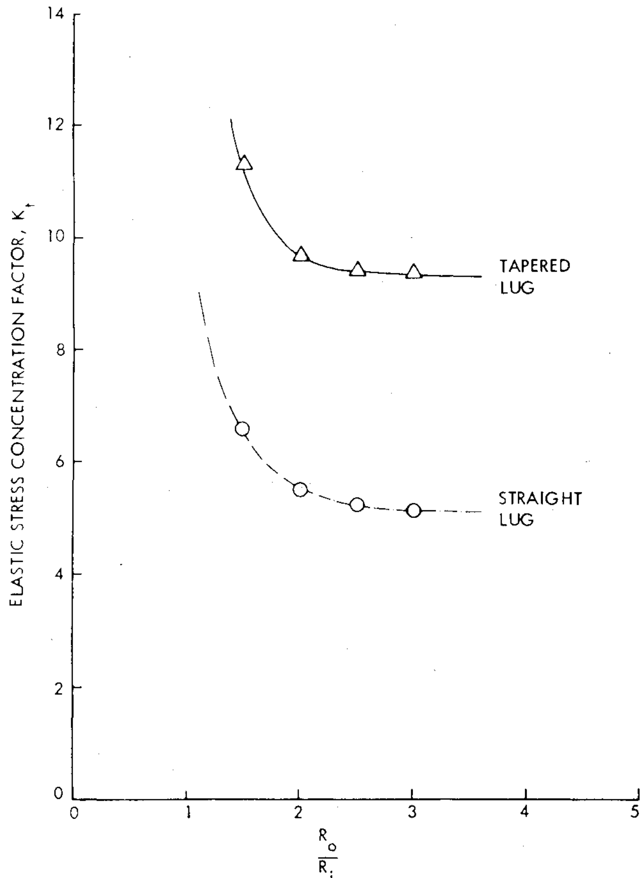

Fig. 11 Elastic stress concentration factors for straight and tapered attachment lugs.

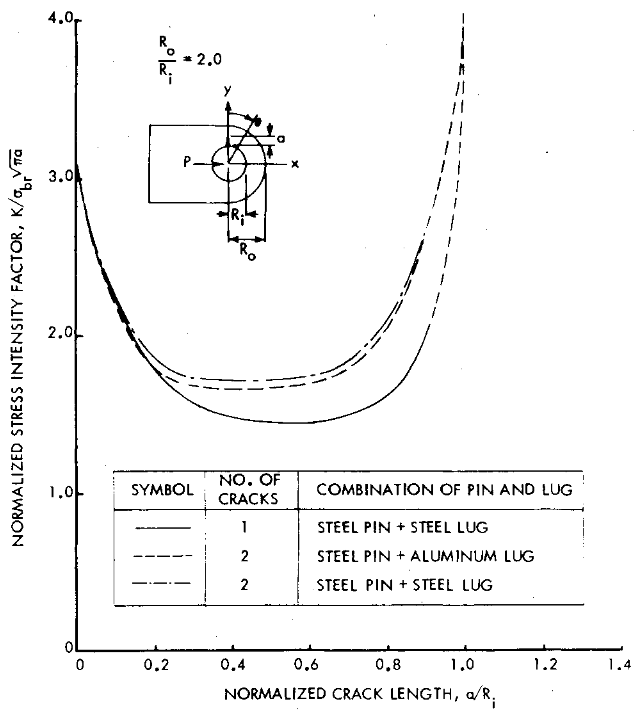

Fig. 12 Normalized stress intensity factors for cracks emanating from straight aluminum and steel lugs having $R_o/R_i = 2.0$.

lugs. At the edge of the hole ($a/R_i = 0$), the normalized stress intensity factor was obtained by multiplying the stress concentration factor determined from unflawed stress analysis by 1.12, which was derived by Gross et al.[21] for a straight edge crack in a finite-plate specimen loaded in tension. Unflawed elastic stress distributions on the prospective crack plane were also obtained and presented in Figs. 9 and 10, for straight and tapered lugs having various R_o/R_i ratios, respectively. The corresponding stress concentration factors (expressed in terms of the far-field gross section stress) K_t at the edge of the hole are given in Fig. 11. As shown in this figure, the stress concentration factor increases with the decrease of R_o/R_i ratio. For example, for straight lugs, the K_t factor increases from 5.15 for $R_o/R_i = 3.0$ to 6.58 for $R_o/R_i = 1.5$. From Figs. 7 and 8, it is evident that the stress intensity factors obtained for a tapered lug are smaller than those obtained for a straight lug. This may be caused by the larger volume of material on the projected crack plane of the tapered lug.

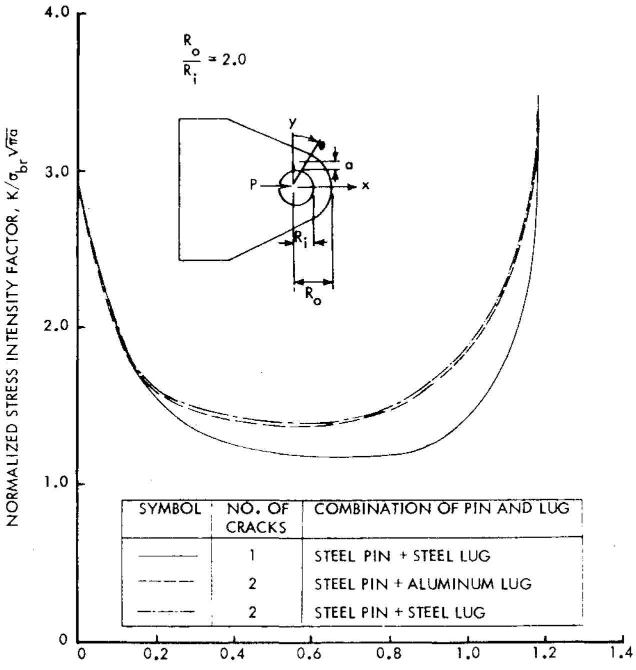

Fig. 13 Normalized stress intensity factors for cracks emanating from tapered aluminum and steel lugs having $R_o/R_i = 2.0$.

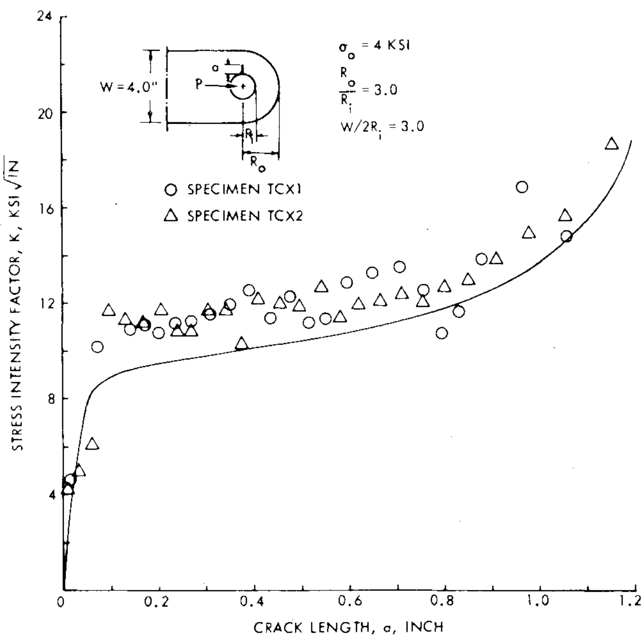

Fig. 14 Stress intensity factors for a single through crack emanating from a straight aluminum lug, $\sigma_0 = 4.0$ ksi.

A typical comparison of K value obtained for a single crack and a symmetrical double crack in a straight steel lug loaded by a steel pin is shown in Fig. 12. A similar result obtained for a tapered lug is presented in Fig. 13. As anticipated, the computed K value for a double crack is higher than that of a single crack. The percentage increase in K value increases with the crack length. It increases from less than 1% for $a/R_i \leq 0.2$ to about 30% for $a/R_i = 0.9$. To study the effect of the relative rigidity of the pin and the lug on the stress intensity factor, the computed K value for a double crack emanating from the hole wall of an aluminum lug loaded by a steel pin is also included in Figs. 12 and 13. As presented in these figures, the K value computed for a combination of the steel pin and the steel lug is slightly higher than that of the steel pin and the aluminum lug, but the difference is practically negligible.

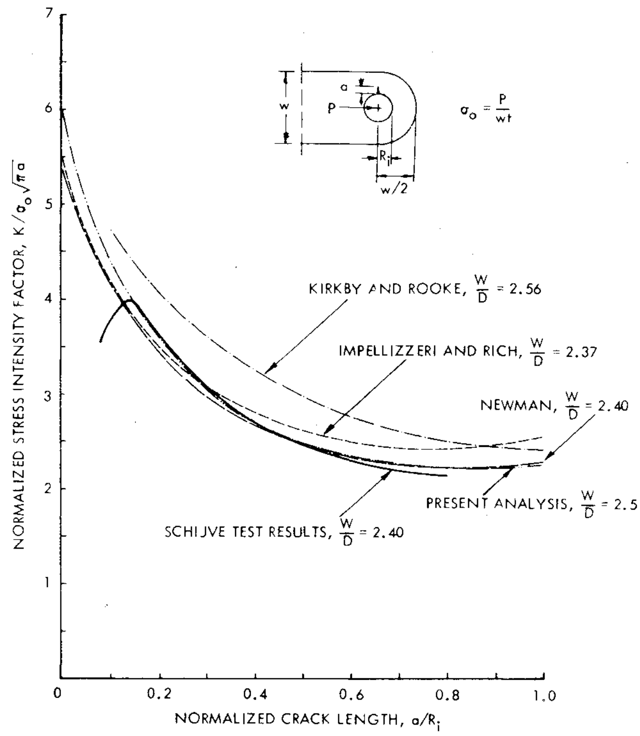

Fig. 15 Correlations of the experimental stress intensity factors and various theoretical predictions.

To assess the accuracy of the computed stress intensity factors, two fatigue crack growth tests were conducted on straight lugs of 2219-T851 aluminum plate material under constant-amplitude loading with a maximum gross section stress of 4 ksi and a stress ratio of 0.1. The specimens had a width-to-hole diameter ratio of 3.0. The diameter of the hole was 1.3330 in., and the hole was loaded by a 4340 steel pin. The diametral clearance of the fitting was about ± 0.0005 in. The thicknesses of specimens were 0.5 and 0.25 in., respectively. The test was performed under room temperature, lab air condition. During the test, specimens were loaded in an electro-hydraulic servo controlled closed loop test machine operated at 10 cycles/s using a fork and pin arrangement and hydraulic grip. The fork and pin were precision machined to assure the required symmetry and fit. Periodically, the pin was removed and the actuator ram lowered to permit visual crack-length measurements. Aluminum deposits on the pin surface resulting from fretting were removed using 2/0 grit polishing paper. The crack lengths observed on each surface of the lug were essentially equal; however, the average value was used to reduce the stress intensity factors using the fatigue crack growth method of calibrating the crack growth rate da/dN, and the stress intensity factor range ΔK. The da/dN vs ΔK data was generated under the same test conditions using a compact specimen. The reduced K values are presented in Fig. 14 as circular symbols for a 0.5-in.-thick specimen and as triangular symbols for a 0.25-in.-thick specimen. The computed stress intensity factors, represented by the curve, are also included in the figure. The computed K values are about 10-18% below those reduced from test data. One of the explanations is that, during these two tests, the pin was periodically removed for crack-length measurement, and aluminum deposits on the pin surface resulting from fretting were cleaned; this resulted in a slightly loose fit between the pin and the lug. As mentioned previously, the effect of pin fitting on the stress intensity factor is currently under study. Some initial results indicated that, for a constant crack length, an increase in the clearance between the pin and the lug increases the stress intensity factor.

The current analysis is also used to correlate with experimental K values reported in Ref. 10. These K values were reduced from crack growth tests on 2024-T3 aluminum lugs loaded by an alloy steel pin using the same procedure. During these tests, the pin had never been removed and cleaned, and crack growth observations were made through the slot made in the steel fork. This test condition is much closer to the assumed neat-fit condition used in the present analysis. The results of correlations between the current analysis and these test data are shown in Fig. 15. The predictions using the simple compounded solution method by Kirkby and Rooke[14] and Newman[22] and the modified weight function method by Impellezzeri and Rich[18] are also included in the figure. It is clear that the current analysis using the cracked finite-element method gives the best correlation with the data generated by Schijve and Hoeymakers.[10]

Conclusions

The finite-element method with the inclusion of a high-order crack-tip singularity element has been used to determine the pin-bearing pressure distributions and the stress intensity factors for cracks in straight and tapered attachment lugs. The current analysis accurately can account for the change of pin-bearing pressure distribution with the change in crack length, and provides K values which are in good agreement with the available data. Based on the analysis conducted, the following conclusions have been reached.

1) The pressure distribution between the pin and the lug changes with the change in crack length. In the presence of a crack, the pressure distributions change significantly compared with the uncracked case.

2) For a constant normalized crack length a/R_i and a constant applied load, the variation of pin-bearing pressure distribution and the corresponding stress intensity factor are most significant when the outer-to-inner radius ratio R_o/R_i is small. When the R_o/R_i ratio increases, the variation in the pressure distribution and the computed stress intensity factor decrease.

3) For a constant R_o/R_i and a constant load applied at the pin, the tapered lug provides a lower stress concentration (in terms of average bearing stress), lower stress intensity factors, and less pin-bearing pressure variation than the straight lug.

4) Although the stress intensity factor computed for a combination of the steel pin and the steel lug is slightly higher than that of the steel pin and the aluminum lug, the difference is practically negligible.

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